Waves on the Web: an On-Line One-Dimensional Hydrodynamics Simulator

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Abstract

The one-dimensional hydrodynamics simulator provides an on-line interface for pedagogical purposes, allowing students to study the effects of the equations of motion for fluid dynamics in one-dimension. It is part of a larger project, the Digital Demo Room, that aims to provide several different simulators of interest to astrophysical applications.

1. Background and Introduction

1.1. Project Motivation

The original computer code for the one-dimensional hydrodynamics simulator was written by Professor Charles F. Gammie to assist in basic fluid dynamics instruction. Although a useful tool, the code was difficult to apply to a classroom environment in its original format. Learning to use the software required to propagate and plot the waves detracted from the primary goal of teaching basic fluid dynamics. For this reason the one-dimensional hydrodynamics simulator was developed and added to the Digital Demo Room, providing an interactive on-line user interface that puts the workings of the code and plotting programs in the background and allows the student to concentrate on learning the physics instead of the programming. The simulator takes the user's inputs for various physical and computational parameters and performs all the necessary calculations to produce an animated plot of the wave in Motion Picture Experts Group (mpeg) movie format.

In its present form, the simulator is designed to propagate linear waves, which may or may not be under the influence of magnetic fields, and hydrodynamic shock tubes. The absence of any magnetic field influence defines hydrodynamic waves; alternatively, effects due to magnetic fields result in more complicated wave motions that fall under the heading of magnetohydrodynamics (MHD).

1.2. One Dimensional Hydrodynamics

To simulate hydrodynamic and MHD waves, the simulator uses formulas derived from the basic conservation equations of fluid dynamics and Maxwell's equations for magnetic fields:

Mass conservation (continuity equation):

$$\frac{\partial \rho}{\partial t} = -\boldsymbol{\nabla} \cdot (\rho \boldsymbol{v})$$

Momentum conservation:

$$\frac{D\boldsymbol{v}}{Dt} = -\frac{\boldsymbol{\nabla} p}{\rho} - \boldsymbol{\nabla}\Phi - \frac{\boldsymbol{\nabla}\left(B^2\right)}{8\pi\rho} + \frac{\left(\boldsymbol{B}\cdot\boldsymbol{\nabla}\right)\boldsymbol{B}}{4\pi\rho}$$

Energy conservation:

$$\frac{Du}{Dt} = -\left(\gamma u \boldsymbol{\nabla}\right) \cdot \boldsymbol{v}$$

Magnetic Induction:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B})$$

Divergence-Free Formula:

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$$

In these equations, ρ, $\boldsymbol{v}$, p and u represent the fluid density, velocity, pressure and internal energy respectively, t stands for time and $\boldsymbol{B}$ is the external magnetic field vector. For hydrodynamic waves, only the first three equations are relevant and the two magnetic field terms in the momentum conservation equation (which represent magnetic pressure and tension respectively) are ignored.

$\frac{D}{Dt}$ is called the **convective derivative** defined as

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (\boldsymbol{v} \cdot \boldsymbol{\nabla})$$

and Φ is the gravitational potential given by the **Poisson equation**

$$\nabla^2 \Phi = 4\pi G \rho$$

The energy conservation formula is written in internal energy form rather than total energy.

An additional constraint used is the **ideal gas equation of state**

$$p = (\gamma - 1)\, u$$

which relates the fluid pressure to the internal energy using the adiabatic index γ (see section 2.3.2. Physical Parameters).

The equations are evolved using the ZEUS method (Stone & Norman 1992a,b). A full description of the method is beyond the scope of this paper.

1.2.1. Linear Waves

One part of the simulator allows the user to integrate small amplitude waves. A perturbation is introduced into the conservation and magnetic field equations by modifying the variable terms as follows:

$$\rho = \rho_0 + \delta\rho_{(\boldsymbol{x},t)}$$

$$\boldsymbol{v} = \boldsymbol{v}_0 + \delta\boldsymbol{v}_{(\boldsymbol{x},t)}$$

$$u = u_0 + \delta u_{(\boldsymbol{x},t)}$$

$$\boldsymbol{B} = \boldsymbol{B_0} + \delta\boldsymbol{B}_{(\boldsymbol{x},t)}$$

where ρ_0, $\boldsymbol{v_0}$, u_0 and $\boldsymbol{B_0}$ represent initial conditions of the fluid element and magnetic field and $\delta\rho_{(\boldsymbol{x},t)}$, $\delta\boldsymbol{v}_{(\boldsymbol{x},t)}$, $\delta u_{(\boldsymbol{x},t)}$ and $\delta\boldsymbol{B}_{(\boldsymbol{x},t)}$ signify small changes in the density, velocity, internal energy and magnetic field from the undisturbed equilibrium conditions. To create an initial equilibrium, ρ_0, u_0 and $\boldsymbol{B_0}$ are made constant and $\boldsymbol{v}_0$ is set to zero.

To begin the linearization of the conservation equations, the following forms for the perturbations are adopted:

$$\delta\rho = \delta\tilde{\rho} e^{(i\boldsymbol{k}\cdot\boldsymbol{x} - i\omega t)}$$

$$\delta\boldsymbol{v} = \delta\tilde{\boldsymbol{v}} e^{(i\boldsymbol{k}\cdot\boldsymbol{x} - i\omega t)}$$

$$\delta u = \delta\tilde{u} e^{(i\boldsymbol{k}\cdot\boldsymbol{x} - i\omega t)}$$

$$\delta\boldsymbol{B} = \delta\tilde{\boldsymbol{B}} e^{(i\boldsymbol{k}\cdot\boldsymbol{x} - i\omega t)}$$

Similar perturbations are used for the pressure and potential terms to create factors of $\delta\tilde{\Phi}$ and $\delta\tilde{p}$; however, substitution and linearization of the Poisson equation and the ideal gas equation of state produce the relations $\delta\tilde{\Phi} = -\frac{4\pi G}{k^2}\delta\tilde{\rho}$ and $\delta\tilde{p} = (\gamma - 1)\,\delta\tilde{u}$. These terms can then be substituted back into the conservation equations to eliminate the changes in potential and pressure as variables.

To complete the linearization of the conservation equations the previous substitutions are performed and the equations are simplified, ignoring terms of second order and higher. In addition, since the simulator only only allows quantities to vary along one dimension, the wave vector term ($\boldsymbol{k}$) is set so that $\boldsymbol{k} = k\hat{\boldsymbol{i}}$.

1.2.1.1. Hydrodynamic Waves

For hydrodynamic waves the magnetic pressure and tension terms of the momentum conservation equation are dropped and the magnetic induction and divergence-free formulas are ignored. Using the substitutions described above results in the following linearized equations of motion:

$$i\omega\delta\tilde{\rho} - i\omega\rho_0\delta\tilde{\boldsymbol{v}} = 0$$

$$\frac{4\pi iG\rho_0}{k}\delta\tilde{\rho} + i\omega\rho_0\delta\tilde{\boldsymbol{v}} - ik\left(\gamma-1\right)\delta\tilde{u} = 0$$

$$ik\gamma u_0\delta\tilde{\boldsymbol{v}} - i\omega\delta\tilde{u} = 0$$

With the conservation equations properly linearized and perturbed the coefficients of the equations can be written in matrix form. Evaluating the determinant of this matrix produces the natural frequencies of the waves and their associated eigenvectors:

$$\begin{vmatrix} i\omega & -ik\rho_0 & 0 \\ \frac{4\pi iG\rho_0}{k} & i\omega\rho_0 & -ik\left(\gamma-1\right) \\ 0 & ik\gamma u_0 & -i\omega \end{vmatrix} = 0$$

This determinant yields the relation

$$\omega\left(\omega^2\rho_0 - k^2\gamma\left(\gamma-1\right)u_0 + 4\pi G\rho_0^2\right) = 0$$

which gives three natural frequencies: $\omega = 0$ and $\omega = \pm\sqrt{\frac{k^2\gamma(\gamma-1)u_0}{\rho_0} - 4\pi G\rho_0}$. The sound speed c_s can be defined as $c_s^2 = \frac{\gamma(\gamma-1)u_0}{\rho_0}$, which turns the second two frequencies into the familiar dispersion relation for a gravitationally-modified sound wave: $\omega^2 = k^2c_s^2 - 4\pi G\rho_0$.

Plugging the natural frequencies back into the matrix and solving for the eigenvectors associated with each wave gives the relations:

$$\begin{bmatrix} \delta\tilde{\rho} \\ \delta\tilde{v}_x \\ \delta\tilde{u} \end{bmatrix} = \delta\tilde{\rho}\begin{bmatrix} 1 \\ 0 \\ \frac{4\pi G\rho_0}{k^2(\gamma-1)} \end{bmatrix}$$

for the frequency $\omega = 0$. This frequency/eigenvector combination defines an *entropy wave.* The sound wave frequencies produce the eigenvector:

$$\begin{bmatrix} \delta\tilde{\rho} \\ \delta\tilde{v}_x \\ \delta\tilde{u} \end{bmatrix} = \delta\tilde{v}_x\begin{bmatrix} \frac{k\rho_0}{\omega} \\ 1 \\ \frac{k\gamma p_0}{\omega(\gamma-1)} \end{bmatrix}$$

Linearization of the hydrodynamics equations of motion therefore results in two types of waves: entropy waves and sound waves.

1.2.1.2. Magnetohydrodynamic Waves

Linearization of MHD waves is similar to the procedure outlined for hydrodynamic waves with the addition of the magnetic terms in the momentum equation and the additional constraints of the magnetic induction and divergence-free formulas. As an additional simplification, the value of the magnetic field in the z direction, $\delta\tilde{B}_{z_0}$, is set to zero. Since the wavevector is oriented along the x-axis, this simplification can be made without loss of generality because an arbitrary equilibrium and wave can be rotated into this configuration. The resulting one-dimensional linearized equations are then:

$$-i\omega\delta\tilde{\rho} + i\rho_0k\delta\tilde{v_x} = 0$$

$$-i\omega\delta\tilde{u} + \gamma iu_0k\delta\tilde{v_x} = 0$$

$$-i\omega\delta\tilde{v}_x + \frac{ik\left(\gamma-1\right)}{\rho_0}\delta\tilde{u} - \frac{4\pi iG}{k}\delta\tilde{\rho} + \frac{ikB_y}{\rho_0}\delta\tilde{B}_y = 0$$

$$-i\omega\delta\tilde{v}_y - \frac{ikB_x}{\rho_0}\delta\tilde{B}_y = 0$$

$$-i\omega\delta\tilde{v}_z - \frac{ikB_x}{\rho_0}\delta\tilde{B}_z = 0$$

$$-i\omega\delta\tilde{B}_x = 0$$

$$-i\omega\delta\tilde{B}_y + ikB_y\delta\tilde{v}_x - ikB_x\delta\tilde{v}_y = 0$$

$$-i\omega\delta\tilde{B}_z - ikB_x\delta\tilde{v}_z = 0$$

The inclusion of a magnetic field introduces the possibility of transverse waves, making values for v_y and v_z necessary. Obviously, this greatly complicates the calculations, resulting in an 8×8 matrix that is difficult to evaluate. The determinant of this matrix results in eight natural frequencies. To simplify the equations somewhat the following variables are defined:

$$\boldsymbol{v}_A = \frac{\boldsymbol{B}_0}{\sqrt{4\pi\rho_0}}$$

$$\alpha = \frac{4\pi G\rho_0}{c_s^2k^2}$$

$$\beta = \frac{c_s^2}{v_A^2}$$

The first variable, $\boldsymbol{v}_A$, is called the *Alfven velocity.* The angle θ between the Alfven velocity and the x-axis is also introduced so that $v_{Ax} = \boldsymbol{v}_A \cos(\theta)$ and $v_{Ay} = \boldsymbol{v}_A \sin(\theta)$.

The first natural frequency results in a mode which contains magnetic monopoles and can be eliminated by the constraint $\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$. With the inclusion of the newly defined variables, the remaining seven natural frequencies can be written as:

$$\omega = 0$$

$$\omega = \pm c_s k \sqrt{\beta} \cos(\theta)$$

$$\omega = \pm c_s k \left(\frac{1}{2} \left[1 - \alpha + \beta \pm \left((1-\alpha)^2 + \beta (2 - 2\alpha + \beta) + 4\beta (\alpha - 1) \cos^2(\theta) \right)^{\frac{1}{2}} \right] \right)^{\frac{1}{2}}$$

The $\pm$ terms pull out the remaining frequencies from these three equations.

The first frequency, $\omega = 0$, like the hydrodynamic waves, defines an entropy wave with the eigenvector

$$\begin{bmatrix} \delta\tilde{\rho} \\ \delta\tilde{u} \\ \delta\tilde{v}_x \\ \delta\tilde{v}_y \\ \delta\tilde{v}_z \\ \delta\tilde{B}_x \\ \delta\tilde{B}_y \\ \delta\tilde{B}_z \end{bmatrix} = \delta\tilde{u} \begin{bmatrix} \frac{\gamma-1}{\alpha c_s^2} \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The next two frequencies (the second equation with both + and - terms) represent transverse *Alfven waves.* The eigenvector for the Alfven waves is:

$$\begin{bmatrix} \delta\tilde{\rho} \\ \delta\tilde{u} \\ \delta\tilde{v}_x \\ \delta\tilde{v}_y \\ \delta\tilde{v}_z \\ \delta\tilde{B}_x \\ \delta\tilde{B}_y \\ \delta\tilde{B}_z \end{bmatrix} = \delta\tilde{B}_z \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \pm\frac{1}{\sqrt{4\pi\rho_0}} \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The $\pm$ determines the direction of travel.

The remaining equation, which nests four different frequencies, is much more complicated. Two waves originate from these frequencies: fast and slow waves. The computed values of the frequencies determine which wave is derived; the two frequencies that result in the largest absolute value for ω are fast waves (propagating in two directions) and the two remaining frequencies with smaller absolute values are slow waves traveling in two opposite directions. The eigenvector corresponding to these waves is also complicated, so rather than writing it in matrix form they are given as separate equations. Also, to further simplify the equations the frequency is left written as ω instead of substituting the complicated equation for ω found above. The eigenvector equations are:

$$\delta\tilde{\rho} = \delta\tilde{B}_y \left[\frac{\sqrt{\beta 4\pi\rho_0} \sin\theta}{\alpha c_s} + \left(\frac{\sqrt{4\pi\rho_0}\left(c_s^2 k^2 - \omega^2\right)}{\alpha\omega c_s^2 k} \right) \times \left(\frac{\omega^2 - c_s^2 k^2 \beta \cos^2\theta}{c_s k \omega \sqrt{\beta} \sin\theta} \right) \right]$$

$$\delta\tilde{u} = \delta\tilde{B}_y \left[\frac{c_s \sqrt{4\pi\rho_0}}{(\gamma - 1)\,\omega} \left(\frac{\omega^2 \sin(\theta) - c_s^2 k^2 \beta \cos^2(\theta)}{\sqrt{\beta}\omega \sin(\omega)} \right) \right]$$

$$\delta\tilde{v}_x = \delta\tilde{B}_y \left(\frac{c_s^2 k^2 \beta \cos^2(\theta) - \omega^2}{c_s k \omega \sin(\theta)} \right)$$

$$\delta\tilde{v}_y = \delta\tilde{B}_y \left(\frac{c_s K \sqrt{\beta} \cos(\theta)}{\omega\sqrt{4\pi\rho_0}} \right)$$

$$\delta\tilde{v}_z = 0$$

$$\delta\tilde{B}_x = 0$$

$$\delta\tilde{B}_y = \delta\tilde{B}_y$$

$$\delta\tilde{B}_z = 0$$

In the computer code, factors of $\sqrt{4\pi}$ have been absorbed into the definition of the magnetic field vector $\boldsymbol{B}$ for all the above equations to simplify the programming.

Linearization of the MHD equations of motion therefore results in four types of waves: entropy waves, Alfven waves and fast and slow waves, each with two possible directions of travel.

1.2.2. Shock Tubes

The simulator also allows construction of shock tube initial conditions. A shock tube is a fluid discontinuity, with distinct left and right states on either side of the shock. The simulator allows the user to input values for the fluid properties on each side of the shock.

Although the details of the physics behind shock propagation is beyond the scope of this paper, the default values used by the simulator are taken from a well-known example. The default initial conditions are from the Brio & Wu MHD problem, which is an MHD analog of the Sod hydrodynamic shock tube problem (Stone & Norman 1992b). The values defining the initial conditions of this tube are: $\gamma = 2.0$, $B_{x_0,(\mathrm{left})} = 0.75$, $B_{x_0}(\mathrm{right}) = 0.75$, $B_{y_0}(\mathrm{left}) = 1.0$, $B_{y_0}(\mathrm{right}) = 1.0$, $p_{(\mathrm{left})} = 1.0$, $p_{(\mathrm{right})} = 0.1$, $\rho_{(\mathrm{left})} = 1.0$ and $\rho_{(\mathrm{right})} = 0.125$. Initial fluid velocities for both the left and right state are set to zero. A detailed analysis of this shock tube can be found in Stone & Norman (1991b).

2. The Simulator

The simulator uses a simple web interface. The user is led to the desired wave through a series of questions, which culminates in a page that requests the parameter inputs specific to the type of wave requested. The front page is located at the universal resource locator (url) http://ddr.physics.uiuc.edu/ddr/oned/index.html.

2.1. Knowledge Level Options

Three levels of user knowledge are available: beginner, intermediate and expert. The beginner level interface does not request any user inputs; rather, a list of previously run models is given with descriptions for each and the user simply selects the wave he or she would like to see. At the intermediate level, only a limited number of parameters are requested from the user. None of the computational parameters (see below) are user-defined at this level and only a select number of physical parameters are input by the user. The expert level interface allows the user to define all of the parameters the simulator offers. For purposes of describing the simulator all options available at the expert level are listed in the following paragraphs.

2.2. Wave Type Options

In its present form the simulator only offers two types of waves, as previously described: linear amplitude hydrodynamic and MHD waves and shock tubes.

2.3. Simulation Parameters

Two basic type of parameters are used by the simulator to create movies files: computational and physical. Computational parameters are used by the simulator program to determine properties of the plot itself, while the physical parameters define the properties of the wave being evaluated.

2.3.1. Computational Parameters

Four user-defined computational parameters are available in the simulator at the expert level. All four are requested in both the linear waves and the shock tubes.

2.3.1.1. Boundary Conditions

The boundary conditions tell the simulator what to do with the wave at the edges of the plot. Three options are available: periodic, reflecting and standard. Periodic boundary conditions cause the wave to appear on the opposite side of the graph after it moves past a boundary. With reflecting conditions the wave will “bounce back” from the edges (careful - this condition may cause the wave to “interfere” with itself). Under standard conditions the wave will simply move into a “ghost zone” after passing a boundary.

2.3.1.2. Courant Number

The Courant number helps determine the size of dt, or the magnitude of the timestep between frames. A larger value of the Courant number causes a larger value in dt, and therefore less frames in the final simulation.

2.3.1.3. Dumping Frequency

The simulator creates movie files by first creating a series of still images, one for each timestep in the simulation. Each still image then becomes a single frame in the mpeg movie file. The dumping frequency parameter tells the simulator how often to create an image file. Smaller values of this parameter result in larger numbers of images and will therefore require additional computation time and produce a larger mpeg file. For instance, a value of 0.1 will result in 10 images for every timestep (determined by the "integration time" parameter described next).

2.3.1.4. Integration Time

The integration time parameter determines the length of the time scale run by the simulator, and, therefore, the number of frames in the resulting movie file. A larger value for the integration time will require a longer wait as the simulator performs computations and will also result in a larger movie file. Users with slow internet connections may want to opt for smaller integration times.

2.3.1.5. Numerical Resolution

The numerical resolution determines the "smoothness" of the wave by setting the number of points plotted. The distance scale is divided by the numerical resolution to calculate dx, or the amount of change in "x", for each point plotted. Larger values for this parameter greatly increase the accuracy of the resulting plots; however, larger values also increase the length of time required to perform calculations.

2.3.2. Physical Parameters

The physical parameters define the properties of the fluid and its medium. Which parameters are requested depend on the type of wave the user wishes to evaluate.

2.3.2.1. Adiabatic Index

In an adiabatic process the entropy of each fluid particle remains constant and therefore no heat exchange occurs between different fluid elements. The adiabatic index, denoted γ, describes the specific heat ratios of fluid elements and is used in the simulator according to the equation:

$$p = (\gamma - 1)\, u$$

Both linear waves and shock tubes request inputs for this parameter. The value of γ is given by the relation $\gamma = \frac{n+2}{n}$, where n is the number of degrees of freedom in the fluid element. The default value for γ used by the simulator is $\frac{5}{3}$, appropriate for a monatomic ideal gas.

2.3.2.2. Amplitude

The amplitude parameter determines the maximum and minimum height of the wave. This parameter is only requested for linear waves, and the simulator associates the amplitude value with one of the physical properties of the wave used in its eigenvector and may be different depending on the type of wave selected. For instance, the sound wave eigenvector use the change in velocity in the x direction, $\delta\tilde{v}_x$, but the MHD eigenvectors for fast and slow waves use the change in magnetic field in the y direction, $\delta\tilde{B}_y$, as the independent variable. See the previous section describing each wave's associated eigenvector to determine which physical property is used as the amplitude for each type of linear wave.

2.3.2.3. Artificial Viscosity

Only an ideal fluid or gas will move with no energy lost to the irreversible transfer of momentum from internal friction between fluid elements with differing velocities. The artificial viscosity determines the extent to which this energy is lost during fluid flow.

2.3.2.4. Density

The shock tube simulator allows user inputs for the left and right states of the fluid density. Linear waves have the initial density value preset.

2.3.2.5. Gravitational Potential

On a small scale, gravity has a negligible effect in the fluid equations of motion. However, on large scales such as those found in astrophysical applications gravitational potential can have a dramatic effect (see section 3, Sample Run). As mentioned in section 1, the gravitational potential is given by the Laplace equation:

$$\nabla^2\Phi = 4\pi G\rho$$

This equation allows us to solve for Φ in terms of ρ to eliminate the potential as a variable. However, this substitution introduces the "G" term, or value for the gravitational potential, which is included in the simulator as a user-input value for linear waves (gravitational potential is not an option for shock tubes).

2.3.2.6. Pressure
As with the density, the fluid pressure is a user-input parameter for shock tubes but not linear waves. Shock tube simulations request input for the fluid pressure for both the left and right states.

2.3.2.7. Magnetic Fields
Magnetic fields can have a dramatic effect on a wave composed of charged particles and vice versa. A non-zero value for the magnetic field will affect the wave's velocity according to the "right-hand rule," causing transverse motion that introduces waves not found in simple hydrodynamic problems, such as Alfven and fast and slow waves.

2.3.2.8. Wave Vector
The wave vector (or wave "number" in the case of the simulator since it operates only in one dimension) must be an integer value; any decimal inputs for this parameter will automatically be adjusted to the next lowest integer value. The simulator uses the wave number to calculate the value of "k_x" used by the eigenvectors such that $k_x = 2\pi n$, where n is the wave number input by the user.

2.3.2.9. Velocity
Fluid velocity is another parameter used only in the shock tubes. The velocities in linear waves are all set initially to zero, but shock tubes allow input for both the right and left fluid states.

3. Sample Run: Sound Wave Under Differing Gravitational Potentials

The introduction of a gravitational potential has a dramatic effect on the fluid. For a sound wave, the dispersion relation given by $\omega^2 = c_s^2k^2 - 4\pi G\rho_0$ shows that a value of $\boldsymbol{k}$ exists which can result in a negative value for ω^2 (see Fig. 1). The value of $\boldsymbol{k}$ that makes $\omega^2 = 0$ is called the *Jean's length*, named for its famed discoverer, English mathematician, physicist, and astronomer Sir James Jeans. For simplification, the simulator allows the user to vary the value of G instead of k; the wave number is fixed at 1 so that $k = 2\pi$ (see previous description of the wave vector parameter). In addition, the values of c_s and ρ are both preset to 1 so that solving for G with $\omega^2 = 0$ gives a value of π for the Jean's length. Values of G lower than π in the simulator will therefore result in a stable wave, whereas larger values make the wave unstable.

For the sample run, two sound waves were selected, one a stable wave with $G = 2$ and another unstable with $G = 5$. The stable wave propagates normally; however, the unstable wave quickly causes the fluid density to peak and remain in one position. The next two pages contain still image excerpts from these two runs (Figs. 2 and 3) which were extracted from the middle of each simulation's time sequence. As expected, $G = 2$ resulted in a stable moving wave, but for $G = 5$ the fluid had already reached a density peak before the midpoint of the simulation. The instabilities in the fluid caused by the unstable wave sends shocks through the fluid, causing the chaotic values for v_x in the x velocity plot.

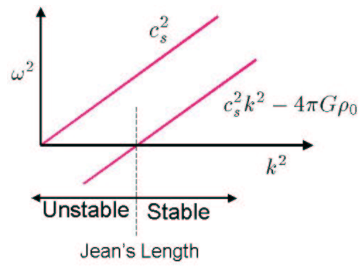

Fig.1: Effect of Jean's length on natural frequency ω

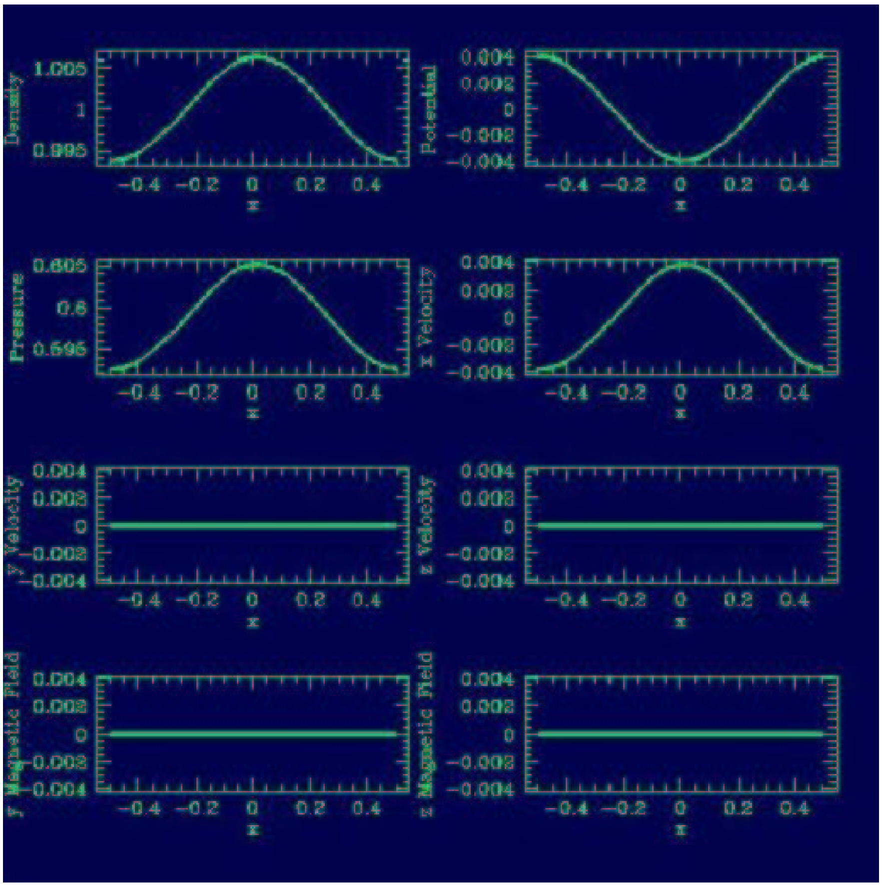

Fig.2: Stable Sound Wave.

Image captured from the middle frame of a sample run of a sound wave with a value of G which is below the Jean's length: $G = 2$. As expected, the simulation created a stable sound wave which propagates through the fluid normally.

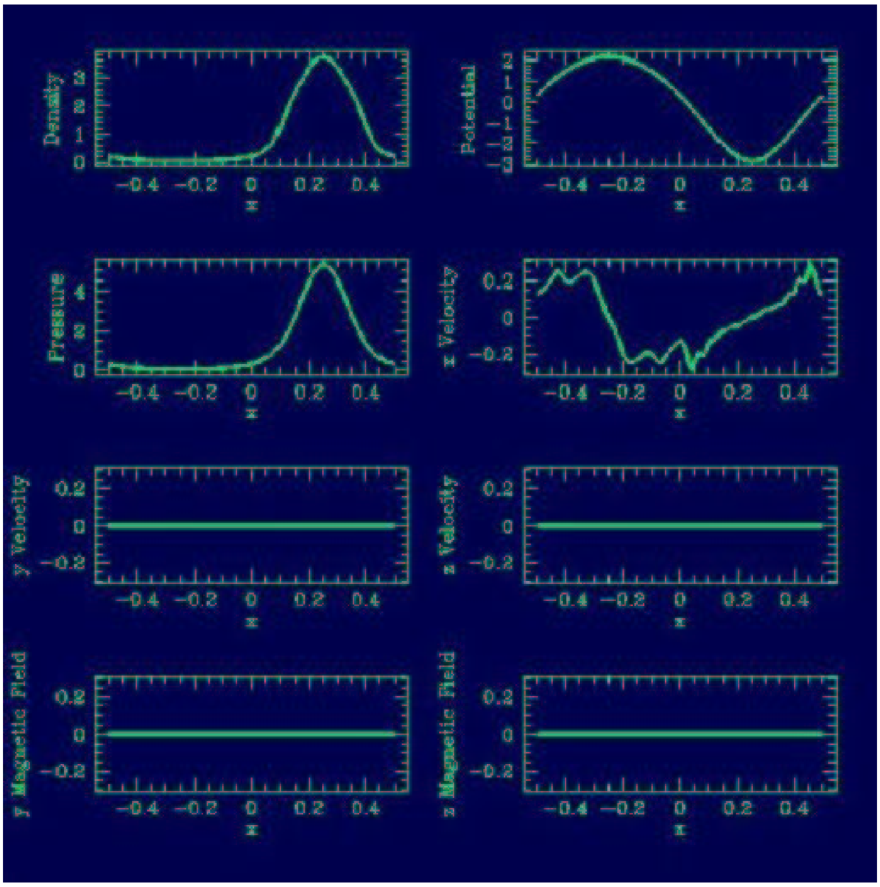

Fig.3: Unstable Sound Wave.

Image captured from the middle frame of a sample run of a sound wave with a value of G which is above the Jean's length: $G = 5$. Also as expected, the simulation created an unstable sound wave which quickly ended in a peak in fluid density.

4. Conclusions

Fluid mechanics, like any scientific field, can be difficult to learn without visualization tools. Development of the One Dimensional Hydrodynamic Simulator, as well as other simulators in the Digital Demo Room, eases instruction of higher-level physics by making available to students and instructors a simulator with an easy to use interactive web-based interface. This interface allows the student to concentrate on learning the physics behind the subject without the inconvenience of having to learn the programming required to create the simulations.

5. Acknowledgments

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